New fundamental discovery of the reverse Fibonacci sequence

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Abstract: New fundamental discovery of the reverse Fibonacci sequence is derived from digital root of Fibonacci sequence. In article is defined the formula of reverse Fibonacci sequence and reverse Fibonacci ratio. It is also shown the basic interpretation and meaning of reverse Fibonacci sequence.

Keywords: Fibonacci, sequence, reverse, ratio,

Introduction

The Golden Ratio and the Fibonacci sequence [1] are well-known mathematical knowledge that humanity has come to know in the earliest times and whose significance is only gradually revealed. The use of the Golden Ratio and the Fibonacci sequence can be observed in nature in different places. The Golden ratio has even been applied in art and architecture, which proves that people feel and are fascinated by the beauty that radiates from the Golden Ratio. In the following article we will expand the knowledge of the Golden Ratio and the Fibonacci sequence and we will get to know that we have so far learned only half of the truth about the Golden Ratio.

Take the known Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, ... that we get by the next member of the sequence as a result of the sum of the two previous members. When we start with the first two members like 1 and 1, the next member is the sum of two, that is 2, the next member is the sumof 1 and 2, that is 3, etc. This step can be repeated any time until infinity. The Fibonacci sequence is an increasing sequence, so the numbers of the Fibonacci sequence increase to infinity. The ratio of the two following members of the Fibonacci sequence approximates to the known Golden Ratio φ.

Reverse Fibonacci sequence

Let's look at the numerical sequence we get when we make a sum of the Fibonaccio numbers and the digits of the Fibonacci number (in following table f.e. number 987=9+8+7=24 and than 2+4=6). It is a great surprise to see that the new line created by this is a repetition of a 24-digit sequence. Let us only consider this 24-digit sequence and the first 24 Fibonacci numbers from the Fibonacci sequence. We see that the first 24 numbers of the Fibonacci sequence are increasing, and each number of the Fibonacci sequence corresponds to a single number of the sum of digits of the given number. Let's ask a question now. Is there a sequence of numbers that would suit the sum of digits of a given number according to the Fibonacci sequence but in reverse order? Yes, a new number sequence can be created that matches the reverse order of the sum of Fibonacci sequence digits. The situation is illustrated in Figure No.1

this new sequence is not random but is given by the formula

$$J_{(n+2)}=8*(J_{(n+1)}-J_n)$$

assuming that the first two members of the sequence are 0 and 1. Zero is the first member because for the sum of figures algorithm it is possible to mix 9 and 0 with the same result for adding digits in numbers.

The sequence derived from the reverse sequence of the sum of the digits of the first 24 numbers of the Fibonacci sequence and created by this formula I call the reverse Fibonacci sequence.

Fibonacciho sequence

Cosmic sequence
(reverse Fibonacci sequence)

1.	1		= 1 =		
2.	1		= 1 =		
3.	2		= 2 =		
4.	3		= 3 =		
5.	5		= 5 =		
6.	8		= 8 =		
7.	13	(1+3)	= 4 =		
8.	21	(2+1)	= 3 =		etc.
9.	34	(3+4)	= 7 =	(5+7+8+4+0+9+2+0+1+6+6+4)	578409201664
10.	55	(5+5)	= 1 =	(8+4+7+0+6+0+6+6+4+3+2)	84706066432
11.	89	(8+9)	= 8 =	(1+2+4+0+4+9+1+6+2+2+4)	12404916224
12.	144	(1+4+4)	= 9 =	(1+8+1+6+6+5+7+9+2+0)	1816657920
13.	233	(2+3+3)	= 8 =	(2+6+6+0+4+3+3+9+2)	266043392
14.	377	(3+7+7)	= 8 =	(3+8+9+6+1+1+5+2)	38961152
15.	610	(6+1+0)	= 7 =	(5+7+0+5+7+2+8)	5705728
16.	987	(9+8+7)	= 6 =	(8+3+5+5+8+4)	835584
17.	1597	(1+5+9+7)	= 4 =	(1+2+2+3+6+8)	122368
18.	2584	(2+5+8+4)	= 1 =	(1+7+9+2+0)	17920
19.	4181	(4+1+8+1)	= 5 =	(2+6+2+4)	2624
20.	6765	(6+7+6+5)	= 6 =	(3+8+4)	384
21.	10946	(1+0+9+4+6)	= 2 =	(5+6)	56
22.	17711	(1+7+7+1+1)	= 8 =		8
23.	28657	(2+8+6+5+7)	= 1 =		1
24.	46368	(4+6+3+6+8)	= 9 =		0
25.	75025	(7+5+0+2+5)	= 1		
26.	121393	(1+2+1+3+9+3)	= 1		
27.	196418	(1+9+6+4+1+8)	= 2		
	etc.				

Figure No.1

Reverse Fibonacci ratio

In the Fibonacci sequence, when we divide two consecutive Fibonacci sequences, we get an approximation of the Golden Ratio number. If we divide two consecutive members of the reverse Fibonacci sequence we will find that this ratio approximates to the number

j = 6,828427112475 ...

respectively,

$$\frac{1}{j}=0{,}1464466094067...$$

I name the number j the reverse Fibonacci ratio and I denote it by the letter j. The reverse Fibonacci ratio will be calculated exactly as follows

$$j=4+\sqrt{8}=4+\sqrt{2^3}=4+2*\sqrt{2}=6{,}828427112475\ldots$$

The significance of the reverse Fibonacci sequence and the reverse Fibonacci ratio

The meaning of the classical Golden Ratio [2] is known as the ratio of the two parts of a line of size 1 that are to each other the same as the ratio of bigger part to the total length of the lin. Thus when we express it mathematically

a : x = x : (a – x)

where a is the size of a line equal to 1 and x is the size of the larger part of the line. From this ratio we get the equation

$$x^2+ax-a^2=0$$

at a line size a = 1 we get a quadratic equation whose solution is the number of the Golden Ratio

$$x=\frac{1}{2}*(\sqrt{5}-1)=0{,}618033988\ldots$$

That is the meaning of the Golden Ratio, which we can transform into different forms.

The meaning of the reverse Fibonacci sequence and the reverse Fibonacci ratio can be understood from the following example.

Suppose we count the number of mothers that will be multiplied in a binary way. So, at every step, we will count all the mothers, assuming that each mother has one offspring at each step, but it must again be the future mother. So we're going to watch the reproduction of females that give birth to one female at every step. Graphically, such reproduction can be represented as a binary tree and the number of females rises exactly according to formula 2^n . To fulfill the condition of the reverse Fibonacci sequence, we need to correct this sequence in such a way that every third generation the original females, that have established the current generation will die. The situation is illustrated in Figure No.2

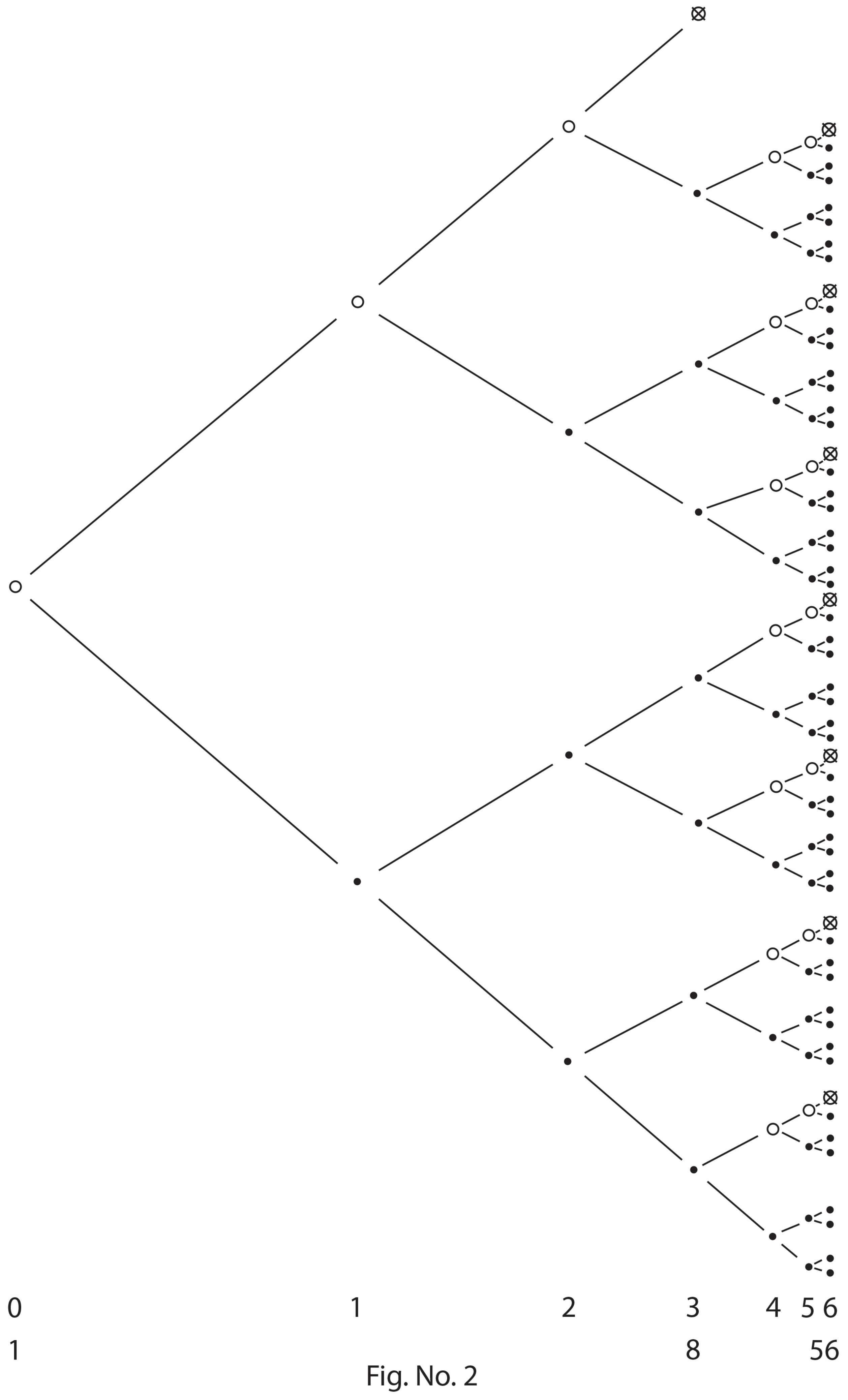

Fig. No. 2

Let's repeat the situation. At the beginning there is one mother-female founder. In the first generation we have the mother of the founder and her daughter. In the second generation, we already have a founding mother and three descendants. In the third generation, we have a founding mother and seven descendants At this moment and step, in the third generation, the founder's mother dies and only her descendants continue to reproduce, and at the time of the founding mother's death, all her descendants become mothers of the founders of the first generation. And so the process is repeated. First generation founding mothers are further reproduced and, after three generations, the descendants of the founder mothers of the first generation die again, and the current descendants become mothers of founders of the second generation.

The growth of the number of the female population where the third generation dies the founding mothers expresses the reverse Fibonacci sequence. The ratio of the growth of this population is expressed by the reverse Fibonacci ratio. Thus, the growth of the female population under these conditions is equal to the exact number of j = 6,828427112475 ...which is about 7 times the population growth for three generations without females dying.

This example we can modify to well known Fibonacciho example with rabbits, where one pair of rabbits give birth in evry step to next one pair of rabbits.

Reverse Fibonacci sequence analysis

As we have already mentioned, the reverse Fibonacci sequence is given by the formula

$$J_{(n+2)}=8*(J_{(n+1)}-J_n)$$

which represents the dying of founders in the third generation. However, what is the situation, if the founders die in the first generation, in the second generation or in the fourth or fifth, etc. We can generalize these considerations and introduce the general concept of the reverse Fibonacci sequence of the x-th order and the general concept of the reverse Fibonacci ratio of the x-th order, which is the expression of the growth of the general reverse Fibonacci sequence of the x-th order. The situation is described in the following table.

Order of reverse Fibonacci sequence (x)	**Formula**	**reverse Fibonacci ratio (growth ratio)**
1	$J_{(n+2)}=2*(J_{(n+1)}-J_n)$	sudden death
2	$J_{(n+2)}=4*(J_{(n+1)}-J_n)$	aproximate to 2
3	$J_{(n+2)}=8*(J_{(n+1)}-J_n)$	6,828427112475...
4	$J_{(n+2)}=16*(J_{(n+1)}-J_n)$	14,9282032302...
5	$J_{(n+2)}=32*(J_{(n+1)}-J_n)$	30,966629547...
...	$J_{(n+2)}=2^x*(J_{(n+1)}-J_n)$	...

Table No. 1

Formula for reverse Fibonacci sequence of the x-th order is

$$J_{(n+2)}=2^x*(J_{(n+1)}-J_n)$$

As can be seen from Table No. 1 The reverse Fibonacci sequence is reverse Fibonacci sequence of the 3th order and reverse Fibonacci ratio is reverse Fibonacci ratio of the 3rd order. Only the reverse Fibonacci sequence fulfills the condition that it is reverse to the Fibonacci sequence according to the procedure described above. The reverse Fibonacci sequence is the first minimal reverse Fibonacci sequence with a constant rate of growth.

Conclusion

The reverse Fibonacci sequence and the reverse Fibonacci ratio was discovered by Slovak engineer Ing. Ondrej Janicko in 2018. The fundamental discovery of the reverse Fibonacci sequence, called the reverse Fibonacci sequence and reverse Fibonacci ratio, is a small but important result of a broader research focusing on the new fundamental foundations of number theory and new fundamental foundations of theoretical physics. The discovery of the reverse Fibonacci sequence and reverse Fibonacci ratio open the new area for research in theoretical mathematics. The significance of the reverse Fibonacci ratio is comparable and has the same strength as other fundamental numbers in mathematics such as the number π, the Euler number e or the classical Golden ratio φ. It is to be expected that new fundamental formulas will be derived, which will contain the number of the reverse Fibonacci ratio and will link it to other fundamental numbers such as, for example, number π or Euler e. The reverse Fibonacci sequence and the reverse Fibonacci ratio is in close relationship with the classic Golden Ratio and the Fibonacci sequence that we come across in nature. The number of reverse Fibonacci ratio is approximately 7. We realize that number 7 is often found in the real world. We have 7 basic colors of rainbow, 7 basic chakras in man, we split time for 7 days a week, etc. The reverse Fibonacci ratio now reveals that it is no coincidence that the number 7 plays such an important role. The number seven embodied in the reverse Fibonacci ratio is now derived directly from mathematical formulas and its basic meaning is understood. It is no coincidence that in real life we see that parents usually do not live longer than the third generation of offspring. As if the nature itself has programmed the length of life. Now it is even confirmed by pure mathematics. So, we can think that the reverse Fibonacci sequence and the reverse Fibonacci ratio are related to the rate of development and growth, with division and flow of time. We can assume that in the future, reverse Fibonacci ratio will be discovered and used in calculations in quantum and nuclear physics, cosmology, chemistry, medicine, biology and others application fields. The classical Golden Ratio is vibrating in number 5. The reverse Fibonacci ratio is vibrating in number 2. 2 + 5 = 7. The discovery of the reverse Fibonacci sequence and reverse Fibonacci ratio has revealed the second half of the truth about the Golden Ratio, that humanity has known until now. Let’s hope that significance of this discovery will be seriously recognized.

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